Chapter - 7 12th CBSE.

~~Ex~~: Integrals.

Indefinite Integrals

diff: $\frac{d}{dx}(F(x)) = f(x)$

In $\int \frac{d}{dx}(F(x)) = \int f(x)\,dx + C$

$$\boxed{F(x) = \int f(x)\,dx + C}$$

properties

(i) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

(ii) $\int k\,f(x)\,dx = k \int f(x)\,dx$

~~(iii)~~ ~~$\int$~~

Eg: $\int (3x^2 + 2x)\,dx$

$= \int 3x^2\,dx + \int 2x\,dx$

$= 3\int x^2\,dx + 2\int x\,dx$

$= 3\left(\frac{x^3}{3}\right) + 2\left(\frac{x^2}{2}\right) + C$

Arithmetic

(iv) $\int dx = x + c$

(or)

$$\int x^0 \, dx = \frac{x^{0+1}}{0+1} + c \qquad x^0 = 1$$

$$= x + c$$

~~trig.~~

(v) $\int \sin x \, dx = -\cos x + c$

(vi) $\int \cos x \, dx = \sin x + c$

~~9/10/2018~~

$$\int \sin ax \, dx = -\frac{\cos ax}{a} + c$$

$$\int \sin(ax+b) \, dx = -\frac{\cos(ax+b)}{a} + c$$

reciproc

$S \rightarrow \text{Cosec}$

$C \rightarrow S$

$\frac{S}{C} \rightarrow \tan$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

XI $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + c$

XII $\int \frac{dx}{\sqrt{1-x^2}} = \cos^{-1}x + c$

XIII $\int \frac{dx}{1+x^2} = \tan^{-1}x + c$

Exponential function

$\int e^x\,dx = e^x + c$

$\int e^{ax}\,dx = \frac{e^{ax}}{a} + c$

$\int e^{(ax+b)}\,dx = \frac{e^{(ax+b)}}{a} + c$

Lograthemic function

$\int \frac{dx}{x} = \log|x| + c$

$\int \frac{dx}{ax+b} = \frac{1}{a}\log|ax+b| + c$

power in x

$\int (a)^{x}\,dx = \frac{a^x}{\log a} + c$

Integrate Ex: 7.1

(i) $\sin 2x$

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$

$$\sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$

$$= \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Anti-derivative $= -\frac{1}{2}\cos 2x$

(X)

$$\int \sin 2x \, dx$$

$$= -\frac{[\cos 2x]}{2}$$

(ii) $\cos 3x$

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x.$$

$$\frac{1}{3}\frac{d}{dx}(\sin 3x) = \cos 3x$$

$$\frac{d}{dx}\left(\frac{1}{3}\sin 3x\right) = \cos 3x$$

(3) (iii) $\frac{d(e^{2x})}{dx} = 2e^{2x}$

$$e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Anti.

4

$(ax+b)^2$

$$\frac{d}{dx}(ax+b)^3 = 3(ax+b)^2 \cdot a$$
$$= 3a(ax+b)^2$$

$$\frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right) = (ax+b)^2$$

$$\int (ax+b)^n dx$$
$$= \frac{(ax+b)^{n+1}}{a(n+1)} + c$$
$$= \frac{1}{3a}(ax+b)^3$$

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$$\sin 2x - 4e^{3x} \longrightarrow (1)$$

$$\frac{d}{dx}(\cos 2x) = 2(-\sin 2x)$$

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right) = \sin 2x.$$

$$\frac{d}{dx}(e^{3x}) = 3e^{3x}$$

$$\frac{d}{dx}\left(\frac{1}{3}e^{3x}\right) = e^{3x}$$

$$(1) \Rightarrow \frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

$$\int \sin 2x \, dx - 4\int e^{3x} dx$$
$$= -\frac{\cos 2x}{2} - 4\frac{e^{3x}}{3} \cancel{+c}$$

(10) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

W.k.T $(a-b)^2 = a^2 - 2ab + b^2$

$\int \left[x - 2\sqrt{x} \cdot \frac{1}{\sqrt{x}} + \frac{1}{x}\right] dx$

$\boxed{\sqrt{2} \cdot \sqrt{2} = 2}$

$= \int \left(x + \frac{1}{x} - 2\right) dx$

$= \int x\, dx + \int \frac{dx}{x} - 2\int dx$

$= \frac{x^2}{2} + \log|x| - 2x + c$

Rough

$\frac{1}{x} = \int \frac{x^{-1+1}}{-1+1}$

$\underline{0}$

(11) $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

$= \int \left(x + 5 - \frac{4}{x^2}\right) dx$

$= \int (x + 5 - 4x^{-2}) dx$

$= \frac{x^2}{2} + 5x - 4 \cdot \frac{(x^{-1})}{-1} + c$

$= \frac{x^2}{2} + 5x + \frac{4}{x} + c$

(12) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{(x^3+3x+4)}{x^{1/2}}\,dx$

$= \int (x^3+3x+4)\,x^{-1/2}\,dx$

$= \int (x^{3-1/2} + 3x^{1-1/2} + 4x^{-1/2})\,dx$

$= \int (x^{5/2} + 3x^{1/2} + 4x^{-1/2})\,dx$

$= \frac{x^{5/2+1}}{5/2+1} + 3\frac{x^{1/2+1}}{1/2+1} + 4\frac{x^{-1/2+1}}{-1/2+1} + c$

$= \frac{x^{7/2}}{7/2} + \cancel{3}\frac{x^{3/2}}{\cancel{3}/2} + \frac{4x^{1/2}}{1/2} + c$

$I = \underline{\frac{2}{7}x^{7/2} + 2x^{3/2} + 8x^{1/2} + c}$

13 $$\int \frac{x^3 - x^2 + x - 1}{(x-1)}\,dx$$

$$= \int \left(\frac{x^2(x-1) + (x-1)}{(x-1)} \right) dx$$

$$= \int \left[x^2 \frac{\cancel{(x-1)}}{\cancel{(x-1)}} + \frac{\cancel{x-1}}{\cancel{x-1}} \right] dx$$

$$= \int (x^2 + 1)\,dx$$

$$= \frac{x^3}{3} + x + C$$

14 $$\int (1-x)\sqrt{x}\,dx$$

$$= \int (1-x)(x^{1/2})\,dx$$

$$= \int (x^{1/2} - x^{1+1/2})\,dx$$

$$= \int (x^{1/2} - x^{3/2})\,dx$$

$$= \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + C$$

$$= \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$$

(15) $\int \sqrt{x}\,(3x^2+2x+3)\,dx$

$= \int x^{1/2}(3x^2+2x+3)\,dx$

$= \int (3x^{5/2}+2x^{3/2}+3x^{1/2})\,dx$

$= 3\cdot\frac{x^{7/2}}{7/2} + 2\frac{x^{5/2}}{5/2} + 3\frac{x^{3/2}}{3/2} + c$

$= \frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + c$

(16) $\int (2x - 3\cos x + e^x)\,dx$

$= \cancel{2}\frac{x^2}{\cancel{2}} - 3\sin x + e^x + c$

(17) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$= \int (2x^2 - 3\sin x + 5(x)^{1/2})\,dx$

$= 2\cdot\frac{x^3}{3} + 3\cos x + 5\frac{x^{3/2}}{3/2} + c$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + c$

18) $\int \sec x(\sec x+\tan x)\,dx$

$= \int \sec^2 x\,dx + \int \sec x \tan x\,dx$

$= \tan x + \sec x + c$

19) $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\,dx$

W.K.T $\sec x = \frac{1}{\cos x}$

$\operatorname{cosec} x = \frac{1}{\sin x}$

$I = \int \frac{1/\cos^2 x}{1/\sin^2 x}\,dx$

$= \int \frac{\sin^2 x}{\cos^2 x}\,dx$

$= \int \tan^2 x\,dx$

W.K.T $\boxed{\sec^2 x = 1+\tan^2 x}$

$\sec^2 x - 1 = \tan^2 x$

$= \int (\sec^2 x - 1)\,dx$

$= \tan x - x + c$

$\int \sin x$

$\int \cos x$

$\int \sec^2 x$

$\int \operatorname{cosec}$

$\int \sec x \tan$

$\int \operatorname{cosec} 1..$

(20) ~~Q.20~~ $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

$= \int \left(\frac{2}{\cos^2 x} - 3 \frac{\sin x}{\cos^2 x} \right) dx$

$= \int \left(2 \sec^2 x - 3 \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \right) dx$

$= \int \left(2 \sec^2 x - 3 \sec x \tan x \right) dx$

$I = 2 \cdot \tan x - 3 \sec x + c$

(21) The anti derivatives of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

$\int \left(x^{1/2} + x^{-1/2} \right) dx$

$\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}$

$= \frac{2}{3} x^{3/2} + 2x^{1/2}$

22 If $\frac{d}{dx}(f(x)) = \int 4x^3 - \frac{3}{x^4}$

s.t $f(2) = 0$ then $f(x)$ is.

$$f(x) = \frac{\cancel{4}x^4}{\cancel{4}} - 3\int x^{-4}\,dx$$

$$= x^4 - \cancel{3}\left(\frac{x^{-3}}{-\cancel{3}}\right) + c$$

$$f(x) = x^4 + x^{-3} + c$$

$f(2)=0$

$$\boxed{f(x) = x^4 + \frac{1}{x^3} + c} \rightarrow (1)$$

$$f(2) = 16 + \frac{1}{8} + c = 0$$

16×8

128

4

$$\Rightarrow \frac{129}{8} + c = 0$$

$$\boxed{c = -\frac{129}{8}}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

Method of Integration

(i) Integration by Substitution

(ii) " using partial fractions

(iii) " by parts.

Type-I

Consider $I = \int f(x)\,dx$

let $\boxed{x = g(t)}$ → (1)

Taking diff on both side, or diff w.r.t t

$$\frac{dx}{dt} = g'(t)$$

x^2

$$\boxed{dx = g'(t)\,dt.}$$

$$I = \int f(x)\,dx$$

$$I = \int f(g(t))\,g'(t)\,dt$$

direct

$$\int (\sin x)\,dx = -\cos x + c$$

$$\int (\cos x)\,dx = \sin x + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x + c$$

✓ $$\int \sec x \tan x\,dx = \sec x + c$$

✓ $$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$$

Special (log terms)

$$\boxed{\int \tan x\,dx = \log|\sec x| + c}$$

$$\int \cot x\,dx = \log|\sin x| + c$$

$$\int \sec x\,dx = \log|\sec x + \tan x| + c$$

$$\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c$$

Ex: 7.2

Q. NO: 1

$$\frac{2x}{1+x^2}$$

$Dr = 1+x^2$

$Nr = \frac{d}{dx}(1+x^2)$

$= 0 + 2x\,dx$

W.k.T

$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

$= 2x^1$

$\int \frac{dx}{x} = \log x$ - formula

$\int \frac{dx}{x} = \log x$

$$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + C}$$

$\int (ax+b)^n dx$

$\int \sqrt{x}\, dx =$

$\int \frac{dx}{\sqrt{x}} =$

$\int (\sqrt{x})^{-1/2} dx =$

Rough

$\frac{(1+x)^{-1+1}}{-1+1}$ $\frac{Any}{0} = \infty$

Soln

$$I = \int \frac{2x}{1+x^2} dx$$

let $\boxed{(1+x^2) = t}$

$2x\,dx = dt$

$(1+x^2)$ ~~dx~~

$2x \frac{dx}{dt} = 1.$

$$I = \int \frac{dt}{t}$$

W.k.T $\int \frac{dx}{x} = \log x + c$

$I = \log t + c$

$I = \log(1+x^2) + c$

Q·NO:2

$$I = \int \frac{(\log x)^2}{x} dx$$

let $\boxed{\log x = t}$

diff on both side.

$$\frac{1}{x} dx = dt.$$

$\boxed{\frac{d}{dx}(\log x) = 1/x}$

$$I = \int (\log x)^2 \frac{dx}{x}$$

$$I = \int t^2 \cdot dt$$

W.K.T $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$$= \frac{t^3}{3} + c$$

$$I = \frac{(\log x)^3}{3} + c$$

Q NO:3

$$I = \int \frac{1}{x + x\log x} dx$$

$$I = \int \frac{1}{x(1+\log x)} dx$$

$(1+\log x) = t$

$$\frac{1}{x} dx = dt$$

$$I = \int \frac{1}{t} dt$$

$$= \int \frac{dt}{t}$$

$$= \log t + c$$

$$\therefore I = \log(1+\log x) + c$$

Q. NO: 4

$I = \int \sin x \sin(\cos x)\, dx$ $\Rightarrow$ inside (bracket) $= t$

let $\cos x = t$

diff $-\sin x\, dx = dt$

$\sin x\, dx = -dt$

$I = \int \sin t\, (-dt)$

$= -\int \sin t\, dt$

$= -[-\cos t] + c$

$= \cos t + c$

$I = \cos(\cos x) + c$

Q. NO: 5

$I = \int \sin(ax+b) \cos(ax+b) \cdot dx$

let $\cancel{ax+b = t}$

$\cancel{a\, dx = dt}$

$\sin\theta \cos\theta$

W.K.T $\sin 2\theta = 2 \sin\theta \cos\theta$

$I = \frac{1}{2} \int 2 \sin(ax+b) \cos(ax+b)\, dx$

$= \frac{1}{2} \int \sin 2(ax+b)\, dx$

W.K.T $\int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a} + c$

$$I = \frac{1}{2}\left[-\frac{\cos 2(ax+b)}{2a}\right] + c$$

$$I = -\frac{1}{4a}\cos 2(ax+b) + c$$

Q. NO: 6

$$I = \int \sqrt{ax+b}\,dx$$

$$I = \int (ax+b)^{1/2}\,dx$$

$\int \cancel{x^n}\,dx =$

$$= \left[\frac{(ax+b)^{3/2}}{a \cdot 3/2}\right] + c \qquad \int (ax+b)^n = \frac{(ax+b)^{n+1}}{a(n+1)}$$

$$= \frac{2}{3a}(ax+b)^{3/2} + c \qquad t^{1/2}$$

Q. NO: 7

$$\int x\sqrt{x+2}\,dx \qquad \int x\left((x+2)\right)^{1/2}$$

$$= \int (x+2-2)\sqrt{x+2}\,dx \qquad \int x^{3/2} + 2x$$

$$= \int (x+2)\sqrt{x+2}\,dx - 2\int \sqrt{x+2}\,dx$$

$$= \int (x+2)'(x+2)^{1/2} dx - 2\int (x+2)^{1/2} dx$$

$$= \int (x+2)^{3/2} dx - 2\int (x+2)^{1/2} dx$$

$$I = \left[\frac{(x+2)^{5/2}}{5/2} \right] - 2\left(\frac{x+2)^{3/2}}{3/2} + c$$

$$\underline{I = 2/5 (x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + c}$$

$\underline{Q \cdot NO}$ $I = \int x\sqrt{1+2x^2}\, dx.$

let $(1+2x^2) = t$

diff on bothsides.

$$4x\, dx = dt$$

$$\boxed{x\, dx = \frac{dt}{4}}$$

$$I = \int \sqrt{t} \cdot \frac{dt}{4}$$

$$= \frac{1}{4}\int (t)^{1/2} dt$$

$$= 1/4 \left(\frac{t)^{3/2}}{3/2} + c\right.$$

$$= \frac{2}{4 \times 3}(t^{3/2}) + c$$

$$= 1/6 \left(\sqrt{(1+2x^2)}\right)^{3/2} + c$$

Q. NO:- 9 $\int (4x+2)\sqrt{x^2+x+1}\,dx$

$= 2\int (2x+1)\sqrt{x^2+x+1}\,dx$

Let $x^2+x+1 = t$

$(2x+1)\,dx = dt$

$= 2\int \sqrt{t}\,dt$

$= 2\int t^{1/2}\,dt$

$= 2\,\dfrac{t^{3/2}}{3/2} + c$

$= \dfrac{4}{3}\,t^{3/2} + c$

$I = 4/3\,(x^2+x+1) + c$

Q. NO:- 10 $\int \dfrac{1}{(x-\sqrt{x})}\,dx$

$= \int \dfrac{1}{\sqrt{x}\cdot\sqrt{x} - \sqrt{x}}\,dx$

$I = \int \dfrac{1}{\sqrt{x}(\sqrt{x}-1)}\,dx \rightarrow (1)$

let $\boxed{\sqrt{x}-1=t}$

$(x)^{1/2}-1=t$

diff on bothsides.

$\frac{1}{2}\cdot x^{1/2-1}\,dx=dt \qquad \frac{d}{dx}(x^n)=nx^{n-1}$

$\frac{1}{2}x^{-1/2}\,dx=dt$

$\frac{1}{2x^{1/2}}\,dx=dt$

$\frac{1}{2\sqrt{x}}\,dx=dt$

$\frac{dx}{\sqrt{x}}=2\,dt$

$① \Rightarrow \quad I=\int \frac{1}{t}\cdot 2\cdot dt$

$=2\int \frac{dt}{t}$

$=2\log t+c$

$I=2\log(\sqrt{x}-1)+c$

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Q. NO $\int \frac{x}{\sqrt{x+4}}\,dx$

$$I = \int \frac{(x+4)-4}{\sqrt{x+4}}\,dx$$

$$= \int \frac{(x+4)^1}{\sqrt{x+4}}\,dx - 4\int \frac{dx}{\sqrt{x+4}}$$

$\sqrt{x}\cdot\sqrt{x} = x$

$\sqrt{(x+4)}\,\sqrt{(x+4)}$

$(x+4)$

$$= \int \frac{(\sqrt{x+4})^{\cancel{2}}}{\cancel{\sqrt{x+4}}}\,dx - 4\int \frac{dx}{\sqrt{x+4}}$$

$$= \int \sqrt{(x+4)}\,dx - 4\int \frac{dx}{(x+4)^{1/2}}$$

$$= \int (x+4)^{1/2}\,dx - 4\int (x+4)^{-1/2}\,dx$$

$$= \frac{(x+4)^{3/2}}{3/2} - 4\,\frac{(x+4)^{1/2}}{1/2} + c$$

$$= \frac{2}{3}(x+4)^{3/2} - 8(x+4)^{1/2} + c$$

$\frac{dx}{(x)^{1/2}}$

$\frac{1}{2}x^{1/2}$

$\frac{1}{2}\sqrt{x}$

$\frac{dx}{x+4}$

$\log(x+4)$

Q. NO: 12

$$\int (x^3-1)^{1/3}\, x^5\, dx$$

$$I = \int (x^3-1)^{1/3} \cdot x^3 \cdot x^2\, dx$$

let $x^3 - 1 = t \Rightarrow \boxed{x^3 = t+1}$

$$3x^2\, dx = dt$$

$$\boxed{x^2\, dx = \frac{dt}{3}}$$

$\boxed{x^3 \cdot x^2 = x^5}$

$$I = \int (t)^{1/3} \cdot (t+1)\, \frac{dt}{3}$$

$$= \frac{1}{3}\left(\int t^{1/3} \cdot t^1\, dt + \int t^{1/3}\, dt\right)$$

$$= \frac{1}{3}\left(\int t^{1/3+1}\, dt + \int t^{1/3}\, dt\right)$$

$$= \frac{1}{3}\int t^{4/3}\, dt + \int t^{1/3}\, dt)$$

$$= \frac{1}{3}\left[\frac{t^{4/3+1}}{4/3+1} + \frac{t^{1/3+1}}{1/3+1}\right] + C$$

$$= \frac{1}{3}\left[\frac{t^{7/3}}{7/3} + \frac{t^{4/3}}{4/3}\right] + C$$

$$= \frac{1}{7}t^{7/3} + \frac{1}{4}t^{4/3} + C$$

$$= \frac{1}{7}(x^3-1)^{7/3} + \frac{1}{4}(x^3-1)^{4/3} + C$$

Q.No

$$I = \int \frac{x^2}{(2+3x^3)^3}\,dx$$

let $2+3x^3 = t$

$$9x^2\,dx = dt$$

$$\boxed{x^2\,dx = \frac{dt}{9}}$$

$$I = \int \frac{1}{t^3}\cdot\frac{dt}{9}$$

$$= \frac{1}{9}\int t^{-3}\,dt$$

$$= \frac{1}{9}\left[\frac{t^{-2}}{-2}\right] + C$$

$$= -\frac{1}{18}\,t^{-2} + C$$

$$= -\frac{1}{18}\cdot\frac{1}{t^2} + C$$

$$= -\frac{1}{18}\cdot\frac{1}{(2+3x^3)^2} + C$$

Q. NO:14

$$I = \int \frac{1}{x(\log x)^m} dx \; ; \; x > 0$$

$\log x = t$

$$\boxed{\frac{1}{x} dx = dt}$$

$\log 0 = 1$

$\frac{1}{0} = \infty.$

$$I = \int \frac{1}{t^m} \cdot dt$$

$$= \int t^{-m} dt.$$

$$= \left[\frac{t^{-m+1}}{-m+1} \right] + c$$

$$\boxed{\frac{(\log x)^{-m+1}}{-m+1} + c}$$

$$= \frac{t^{1-m}}{1-m} + c$$

$$I = \frac{(\log x)^{1-m}}{1-m} + c$$

Q.NO $\int \frac{x}{9-4x^2} dx$

let $9-4x^2 = t$

$-8x\,dx = dt$

$x\,dx = \frac{1}{-8} dt$

$\boxed{x\,dx = -\frac{1}{8} dt}$

$I = \int \frac{1}{t} \cdot \left(-\frac{1}{8}\right) dt$

$= -\frac{1}{8} \int \frac{dt}{t}$.

$= -\frac{1}{8} \log t + C$

$I = -\frac{1}{8} \log(9-4x^2) + C$

Q.No 16 : $\int e^{2x+3} dx$

W.K.T $\int e^{(ax+b)} dx = \frac{e^{ax+b}}{a} + c$

$\int e^{ax} dx = \frac{e^{ax}}{a} + c$

~~direc~~ Subst

$2x+3 = t$

$2\,dx = dt$ $\boxed{dx = \frac{1}{2}\,dt}$

$$I = \int e^{2x+3}\,dx$$

$$= \frac{1}{2}\int e^{t}\cdot dt$$

$$= \frac{1}{2}\left[e^{t}\right] + c$$

$$= \frac{1}{2}\left[e^{(2x+3)}\right] + c$$

direct (Another way)

W.K.T

$$\int e^{(ax+b)}\,dx = \frac{e^{ax+b}}{a} + c$$

$$\int e^{2x+3}\,dx = \frac{e^{2x+3}}{2} + c$$

Trigonometic Identities

Basic Identity

(1) $\boxed{\sin^2 A + \cos^2 A = 1}$

(i) $\sin^2 A = 1 - \cos^2 A$

(ii) $\cos^2 A = 1 - \sin^2 A$

(2) $\boxed{\sec^2\theta - \tan^2\theta = 1.}$

(i) $\sec^2\theta = 1 + \tan^2\theta$

(ii) $\tan^2\theta = \sec^2\theta - 1$

(3) $\boxed{\operatorname{cosec}^2\theta - \cot^2\theta = 1}$

(i) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

(ii) $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

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(1) $\sin(A+B) = \sin A \cos B + \cos A \sin B$ → p

$\sin(A-B) = \sin A \cos B - \cos A \sin B$ -22

$\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$ →23

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ -4

(2) $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

$2 \sin A \sin B = \cos(A+B) - \cos(A-B)$

(3) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

(4) $\sin 2A = 2 \sin A \cos A$.

(ie) $\sin 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

(5) $\cos 2A = \cos^2 A - \sin^2 A$.

(ie) (a) $\cos 2A = 2\cos^2 A - 1$.

(b) $\cos^2 A = \dfrac{1 + \cos 2A}{2}$.

(c) $\sin^2 A = \dfrac{1 - \cos 2A}{2}$

(d) $\cos 2A = 1 - 2\sin^2 A$

(6) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$.

(1) $\sin 3A = 3\sin A - 4\sin^3 A.$

(1) $\cos 3A = 4\cos^3 A - 3\cos A.$

(i) $\sin^3 A = \frac{1}{4}(3\sin A - \sin 3A)$

(ii) $\cos^3 A = \frac{1}{4}(3\cos A + \cos 3A)$

(9) $\tan 3A = \frac{3.\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Q.7 Soln Ex: 7.3

$$\int \sin^2(2x+5)$$

W.k.T $\sin^2 A = \frac{1-\cos 2A}{2}$.

$$I = \int \left(\frac{1-\cos 2(2x+5)}{2} \right) dx$$

$$= \frac{1}{2}\int (1 - \cos(4x+10)) \, dx$$

$$= \frac{1}{2}\left[\int dx - \int \cos(4x+10) \, dx\right]$$

W.k.T $\int \cos(ax+b)dx = \frac{\sin(ax+b)}{a}$.

$$= \frac{1}{2}\left[x - \frac{\sin(4x+10)}{4}\right] + c$$

$$I = \frac{x}{2} - \frac{1}{8}\sin(4x+10) + c$$

Q. No:

$\sin 3x \cos 4x$

let $\boxed{A > B}$

$= \cos 4x \sin 3x$

W.K.T $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

$\int \cos 4x \sin 3x \, dx$

$= \int \frac{1}{2}\left[\sin(4x+3x) - \sin(4x-3x)\right]dx$

$= \frac{1}{2}\left[\int (\sin 7x - \sin x)\, dx\right]$

$= \frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + C$

$= -\frac{\cos 7x}{14} + \frac{1}{2}\cos x + C$

Q. 13

$$\int \cos 2x \left[\cos 4x \cos 6x\right] dx$$

$$= \int \cos 2x \left[\cos 6x \cos 4x\right] dx$$

W.K.T

$$\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$$

$$= \int \cos 2x \left[\frac{1}{2}(\cos 10x + \cos 2x\right] dx$$

$$= \frac{1}{2}\int \left[\cos 2x \cos 10x + \cos 2x \cos 2x\right] dx$$

$$= \frac{1}{2}\int \cos 10x \cos 2x + \frac{1}{2}\int \cos^2 2x \, dx$$

$$= \frac{1}{2}\left[\int \frac{1}{2} \cos(12x) + \cos(8x)\right] + \frac{1}{2}\int\left[\frac{1+\cos 4x}{2}\right]$$

$$\because \cos^2 A = \frac{1+\cos 2A}{2}$$

$$I = \frac{1}{4} \frac{\sin 12x}{12} + \frac{1}{2} \cdot \frac{\sin 8x}{8} + \frac{1}{4}\left[x + \frac{\sin 4x}{4}\right] + C$$

E

(4) $\sin^3(2x+1)$

$I = \int \sin^3(2x+1)\,dx$

W.K.T $\sin^3 A = \frac{1}{4}(3\sin A - \sin 3A)$.

$I = \int \frac{1}{4}\left[3\sin(2x+1) - \sin 3(2x+1)\right]dx$

$= \frac{1}{4}\int 3\sin(2x+1)\,dx - \frac{1}{4}\int \sin(6x+3)\,dx$

$= \frac{3}{4}\cdot\left(-\frac{\cos(2x+1)}{2}\right) + \frac{1}{4}\frac{\cos(6x+3)}{6}\Big] + c$

$I = -\frac{3}{8}\cos(2x+1) + \frac{1}{24}\cos(6x+3) + c$

(5) $\int \sin^3 x \cos^3 x\,dx$

$I = \int (\sin x \cos x)^3\,dx$

W.K.T $2\sin x \cos x = \sin 2x$

$\sin x \cos x = \frac{\sin 2x}{2}$

$I = \int \left(\frac{\sin 2x}{2}\right)^3 dx$

$$= \frac{1}{8}\int \sin^3 2x \, dx$$

W.k.T

$$\sin^3 x = \frac{1}{4}\left(3\sin A - \sin 3A\right).$$

$$= \frac{1}{8\times 4}\left[\int (3\sin 2x - \sin 6x\right] dx$$

$$= \frac{1}{32}\left[\frac{3\cdot(-\cos 2x)}{2} + \frac{\cos 6x}{6}\right] + c$$

$$= -\frac{3\cos 2x}{64} + \frac{\cos 6x}{32\times 6} + c$$

$$= -\frac{3\cos 2x}{64} + \frac{\cos 6x}{192} + c$$

(8) $\int \frac{1-\cos x}{1+\cos x} \, dx$

$$\cancel{\int \frac{(1-\cos x)}{(1+\cos x)} \frac{(1-\cos x)}{(1-\cos x)}} \quad \cancel{= \frac{(1-\cos x)^2}{1^2-\cos^2}} \quad \cancel{= \sin}$$

W.k.T

$$1 - \cos 2A = 2\sin^2 A.$$

$$\boxed{1 - \cos x = 2\sin^2 x/2}$$

$$1 + \cos 2x = 2\cos^2 x$$

$$\boxed{1 + \cos x = 2\cos^2 x/2}$$

$$I = \int \frac{\cancel{2}\sin^2 x/2}{\cancel{2}\cos^2 x/2}\,dx$$

$$= \int \tan^2 x/2 \, dx$$

W.K.T

$$\int \tan^2\theta \, d\theta = (\sec^2\theta - 1)\,d\theta$$

$$= \int (\sec^2 x/2 - 1)\,dx$$

$$= \frac{\tan x/2}{1/2} - x + C$$

$$= 2\tan x/2 - x + C$$

(1) $\dfrac{\cos x}{1+\cos x}$

W.K.T $\cos^2 A = \dfrac{1+\cos 2A}{2}$

$1+\cos 2A = 2\cos^2 A.$

Dr $= \boxed{1+\cos x = 2\cos^2 x/2}$

$\cos 2A = 2\cos^2 A - 1$

Nr $= \boxed{\cos x = 2\cos^2 x/2 - 1}$

$$I = \int \frac{2\cos^2 x/2 - 1}{2\cos^2 x/2}\, dx$$

$$= \int \frac{\cancel{2\cos^2 x/2}}{\cancel{2\cos^2 x/2}}\, dx - \int \frac{dx}{2\cos^2 x/2}$$

$$= \int dx - \frac{1}{2}\int \sec^2 x/2\, dx$$

W.K.T

$$\frac{1}{\cos\theta} = \sec$$

$$= x - \frac{1}{\cancel{2}} \frac{\tan x/2}{1/\cancel{2}} + c$$

$$\underline{I} = x - \tan x/2 + c$$

(10) $\int \sin^4 x\, dx.$

$$= \int (\sin^2 x)^2\, dx$$

W.K.T
$\sin^2 A = \frac{1 - \cos 2A}{2}$

$$= \int \left(\frac{1 - \cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int 1 - 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\left[\int dx - 2\int \cos 2x\, dx + \int \cos^2 2x\, dx\right]$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\cos^2 2x = \frac{1+\cos 4x}{2}$$

$$I = \frac{1}{4}\left[x - \cancel{2}\,\frac{\sin 2x}{\cancel{2}} + \int \left(\frac{1+\cos 4x}{2}\right) dx\right]$$

$$= \frac{1}{4}\left[x - \sin 2x + \frac{1}{2}\left[x + \frac{\sin 4x}{4}\right]\right] + c$$

$$= \frac{x}{4} - \frac{\sin 2x}{4} + \frac{x}{8} + \frac{1}{32}\sin 4x + c$$

$$= \frac{2x + x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$$

$$= \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c$$

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$$\frac{\sin^2 x}{1+\cos x} = \frac{1-\cos^2 x}{1+\cos x}$$

W.K.T $\Rightarrow a^2-b^2 = (a+b)(a-b)$

$$= \frac{\cancel{(1+\cos x)}(1-\cos x)}{\cancel{(1+\cos x)}}$$

$$= \int (1-\cos x)\,dx$$

$$= x - \sin x + C$$